Advanced Systems Lab

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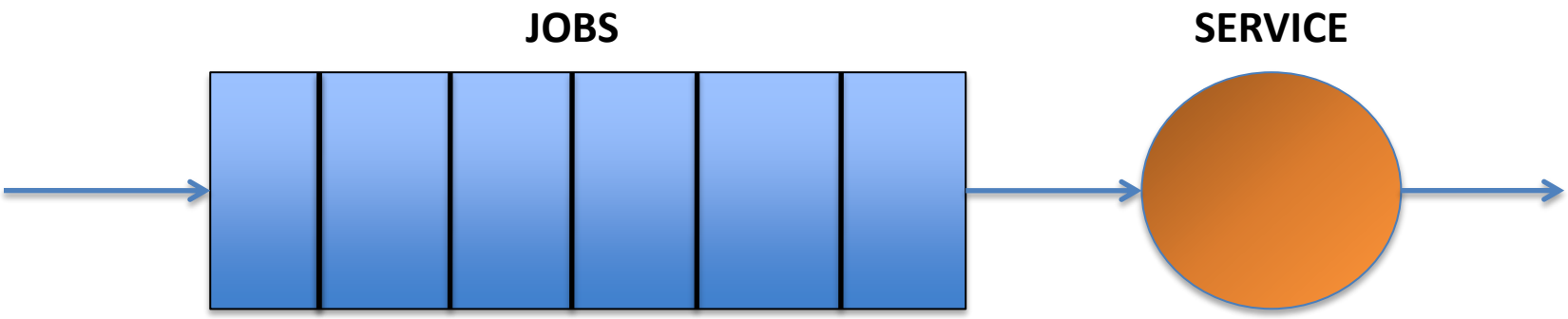
A queue



-Very useful model for many computer systems
- Basic principles of queuing theory
- Understand the limitations of queues as models

Characterizing a queuing system

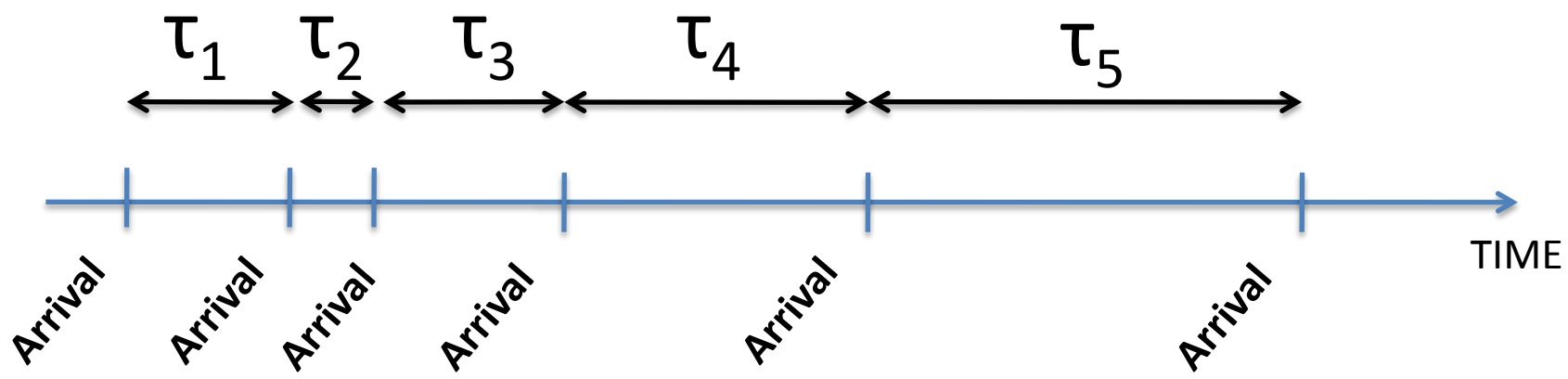
- Arrival rate
- Service time
- Service discipline
- System capacity
- Number of servers
- Population size

- A/S/m/C/P/SD
  - A = arrival distribution
  - S = service distribution
  - m = number of servers
  - C = buffer capacity
  - P = population size (input)
  - SD = service discipline

- Notation not standardized

ARRIVAL RATE DISTRIBUTION

Interarrival time



- The interarrival times are assumed to form a sequence of Independent and Identically Distributed (IID) random variables
- Common assumption is a Poisson distribution

Mean arrival rate

- Mean interarrival time = $E[\tau]$
- Mean arrival rate: $\lambda = 1 / E[\tau]$
- λ is not a random variable!
- Examples;
  - a single client submits a query every 200 ms, then λ is 5 queries/second
  - 10 clients submit a query each every 500 ms, then λ is 20 queries/second
  - These are the queries submitted to the system

Assumptions

- Queuing systems assume an arrival rate
  - state independent (does not depend on number of previous jobs)
  - stationary (does not change in time)
- These assumptions do not hold in real systems
  - Burstiness / batch jobs
  - Flash crowds (popularity)
  - Social effects (time of day variant load)

SERVICE RATE DISTRIBUTION

Service time per job

- The time it takes to process a job (only the time it takes to process it, not including the time it has been waiting in the queue) = s
- Mean service rate: $\mu = 1/E[s]$
- If there are *m* servers, mean service rate is $m\mu$
- μ is not a random variable!
- Example:
  - Printer takes on average 20 seconds per job, then $\mu = 0.05$ jobs/second = 3 jobs/minute

Throughput

- Sometimes μ is called the system's throughput
- Careful with the notion of throughput
- This is correct only in some cases
  - There are always jobs ready when a job is finished
  - No overhead in switching to new job
  - All jobs complete correctly
  - Service rate is state independent (does not depend on the number of jobs in the queue)
  - Service rate is stationary (does not change with time)

SERVICE DISCIPLINE

Queue discipline

- FCFS = First Come – First Served
  - Ordered queue
- LCFS = Last Come – First Served
  - Stack
- RR = Round Robin
  - CPU allocation to processes
- RSS = random
- Priority Based

OTHER PARAMETERS

System capacity

- The system (or buffer) capacity is the maximum number of jobs that can be waiting for service
- System capacity includes jobs waiting and jobs receiving service
- In reality = finite
- Analysis = assume infinite capacity
- Finite buffers very important in practice

Number of Servers

- The service can be provided by one or more servers
- Assume work in parallel and independent
- Servers do not interfere with each other
- Total service rate is aggregation of each individual service rate

Population

- The total number of potential jobs that can be submitted to the system:
- Analysis = assume infinite
- In practice:
  - Very large (assume infinite), e.g., number of clicks on a page
  - Finite, number of homework submissions for this lecture
  - Closed systems (output determines input)

GENERAL RESULTS

G/G/1 G/G/m

Offered load

- The offered load or traffic intensity is

$$\rho = \lambda/(m\mu) = \lambda \cdot E[s]/m$$

- The system is stable if

$$\rho < 1 \Rightarrow \lambda < m\mu$$

- In other words, the system is stable if the mean arrival rate (λ) is less than the mean service rate ($m\mu$), otherwise the queue grows without bounds

ρ = 1

- Unless arrivals and service are deterministic and exactly scheduled, ρ =1 does not lead to a steady system
  - Randomness prevents queuing from emptying
  - Server cannot catch up
  - Queue grows without bound
- One way to avoid this is flow control (drop jobs when load to high)

Examples

Instrumentation shows that a disk is serving 50 I/O operations per second and the average I/O time is 10 ms. What is the disk utilization?

$$\rho = \lambda \cdot E[s]/m, \text{ with } m = 1$$

$$\rho = 50 \times 0.010 = 0.5 = 50\%$$

Application A generates about 50 I/O requests/s, if the disk is 85 % utilized, what is the average time needed for every I/O? ... 17ms

Further examples

We have allocated 60% of the disk to one application. If we want to maintain an average response time for every I/O operations of 12 ms, what is the maximum number of I/O requests per second that the application can generate?

$$0.6 = \lambda \cdot 12 \text{ ms} \Rightarrow \lambda = 50 \text{ req/s}$$

Some more notation

- $n = n_s + n_q$, where
  - n is the number of jobs in the system (queue)
  - n_s is the number of job in the service
  - n_q is the number of jobs waiting for service
- $w = w_q + s$, where
  - w is the total time in the system
  - w_q is the time waiting in the queue
  - s is the time in the service
- These are all random variables

Little's Law

- For the queuing system:

$$E[n] = \lambda \cdot E[w]$$

- For the queue

$$E[n_q] = \lambda \cdot E[w_q]$$

- With

$$E[n] = E[n_q] + E[n_s]$$

$$E[w] = E[w_q] + E[s] = E[w_q] + 1/\mu$$

Example

Instrumentation shows that the average time to respond to a request was 100 ms and the server received about 100 requests/second. If each active request requires 5 KB of memory, how much memory needs to be reserved for the average number of requests in the system?

Jobs in the system = $100 \cdot 0.1 = 10$ jobs (Little's)

Memory needed = $10 \cdot 5$ KB = 50 KB

Jobs in service

- Using Little's Law, one can derive:

$$E[n_s] = \lambda/\mu = \lambda \cdot E[s]$$

- For a queuing system with m servers

$$E[n_s] = m \cdot \rho$$

that is, the average number of jobs in service is m times the arrival rate divided by the mean service rate

BIRTH-DEATH PROCESSES

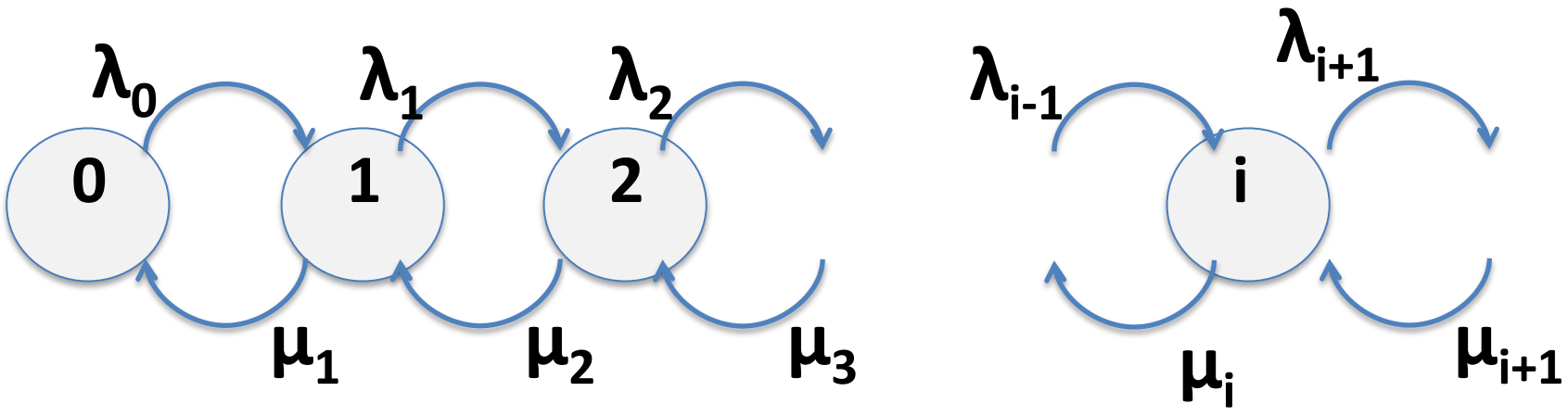
Stochastic processes

- Many of the values in a queuing system are random variables function of time (e.g., the waiting time at a queue)
- Such random functions of times are called stochastic processes
- If the values a process can take are finite or countable, it is a discrete process or a stochastic chain

Markov Processes

- If the future states of a process depend only of the current state and not on past states, the process is a Markov process
- Discrete Markov processes are Markov chains
- A Markov chain in which the transition between states is limited to neighboring states is called a birth-death process

Steady state probability



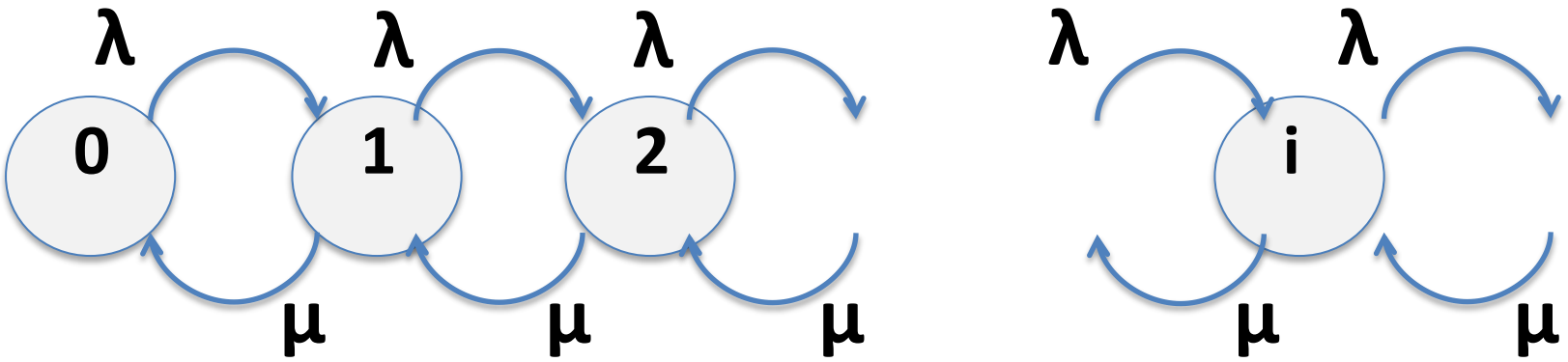
- Probability of being in state n is:

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

M/M/1

M/M/1

- Memoryless distribution for arrival and service
- Single server
- Infinite buffers and FCFS
- Mean arrival rate: λ
- Mean service rate: μ



Basics M/M/1

- From the state probability of a birth-death process:

$$p_n = (\lambda/\mu)^n \, p_0, \quad n = 1,2,\ldots,\infty$$

- or

$$p_n = \rho^n \, p_0$$

- Since all probabilities must add to 1

$$p_0 = 1 - \rho \qquad \text{and} \qquad p_n = (1 - \rho)\,\rho^n$$

Utilization

- Utilization: probability that there is one or more jobs in the system

$$U = 1 - p_0 = \rho$$

M/M/1 behavior

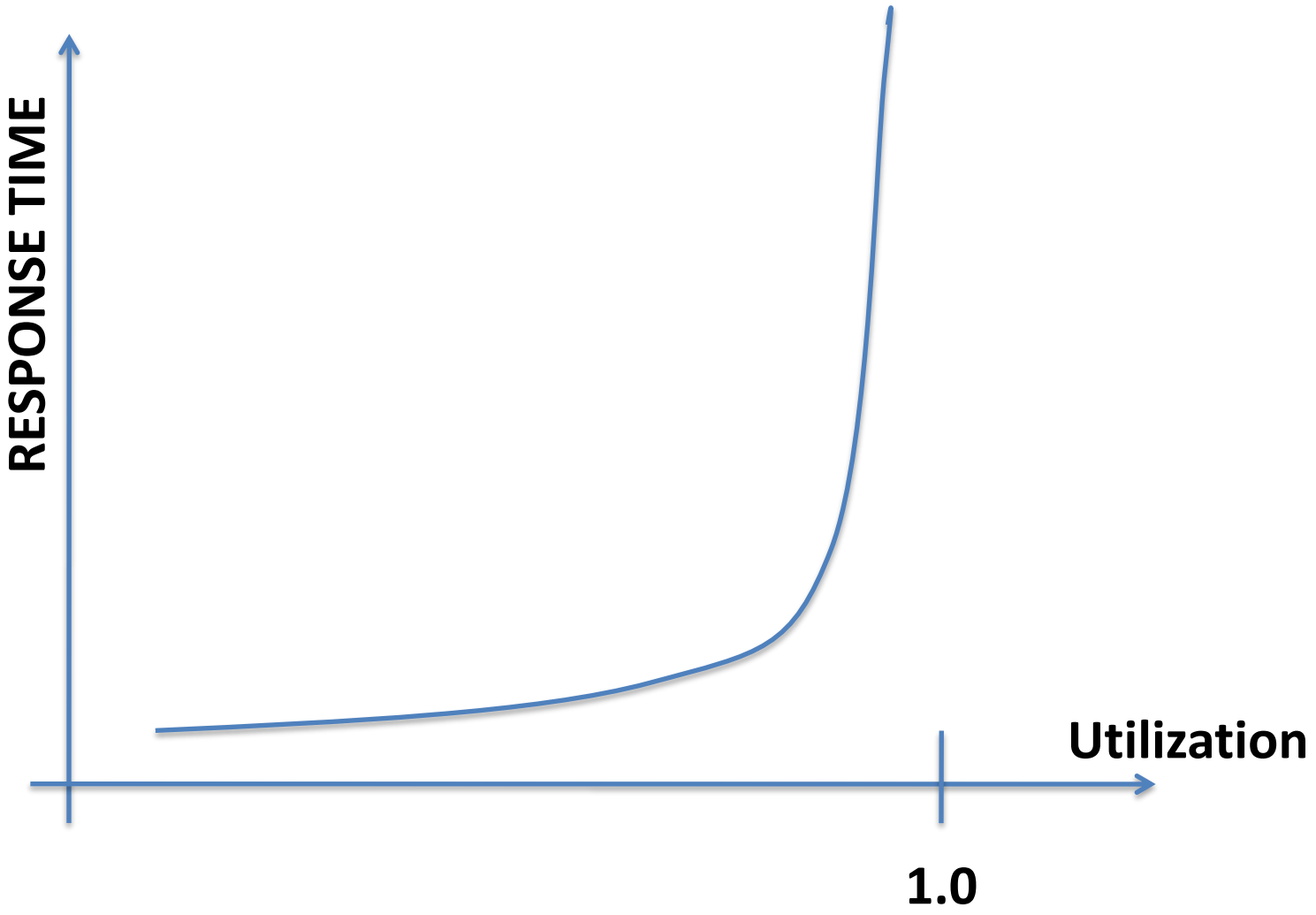
- The mean number of jobs E[n] is

$$E[n] = \sum_{n=1}^{\infty} n \cdot p^n = \sum_{n=1}^{\infty} n(1-\rho)\rho^n = \frac{\rho}{1-\rho}$$

- Applying Little's Law we get the response time

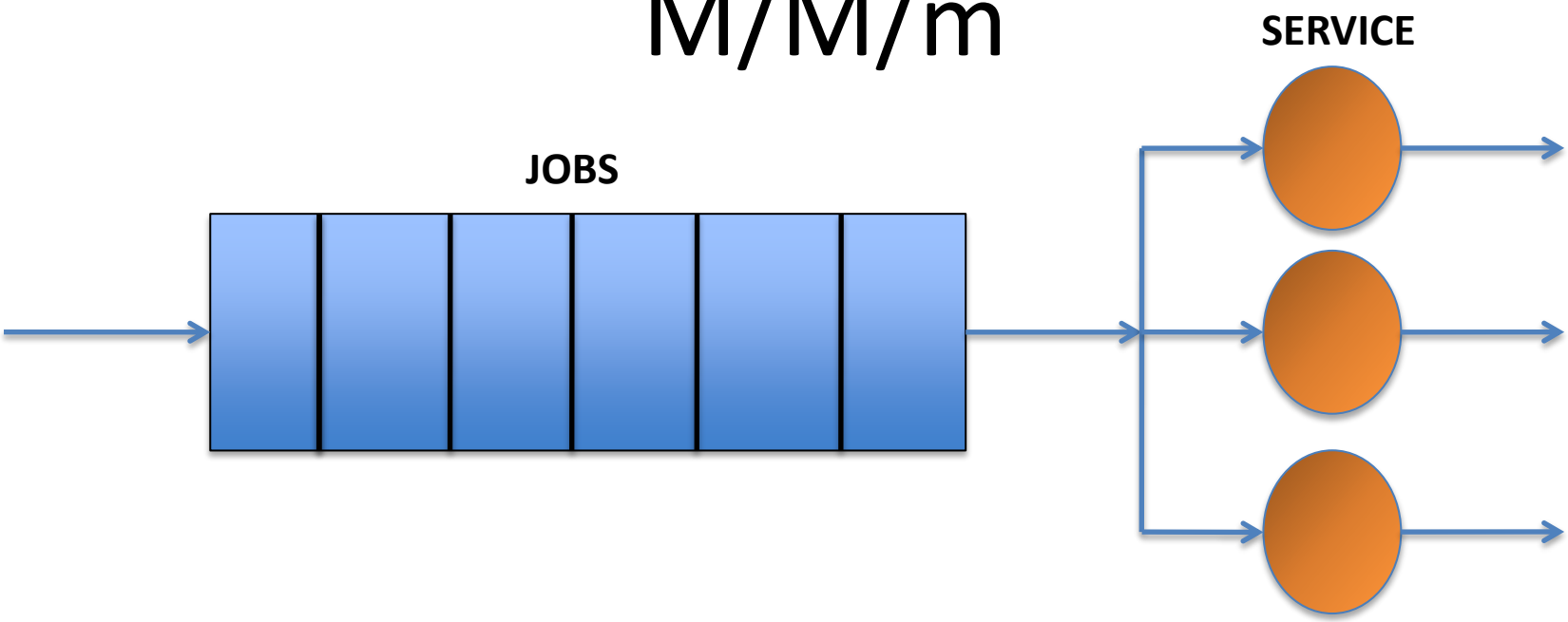
$$E[w] = \frac{1/\mu}{1-\rho}$$

Response time in M/M/1



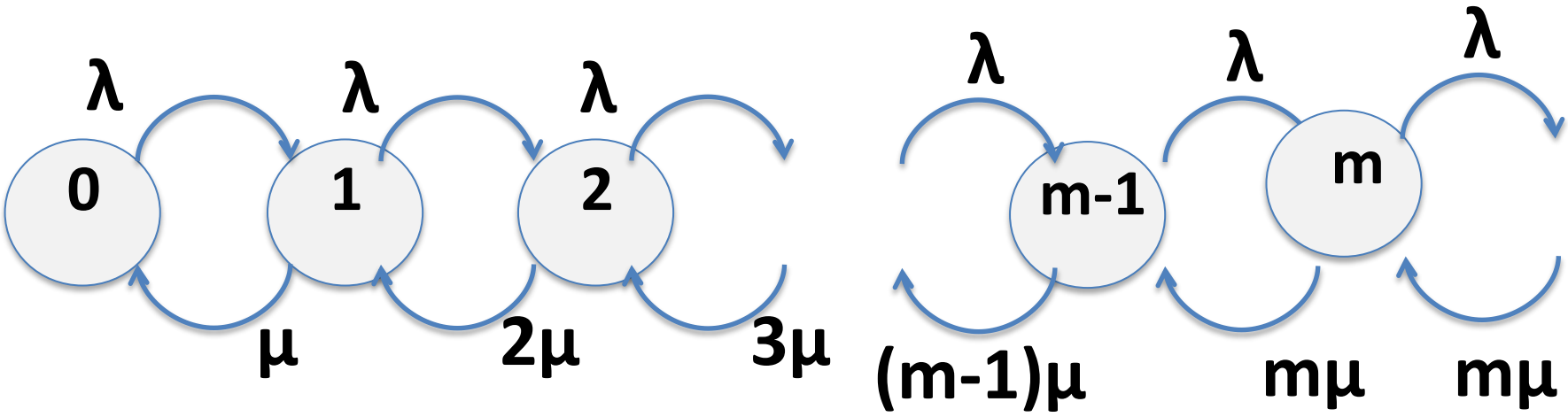
M/M/m and M/M/m/B

M/M/m



- Each server serves μ jobs per unit of time
- Jobs get service right away if less than m jobs in system, otherwise they wait in queue.

Probabilities in M/M/m



- Number of jobs in a M/M/m system is a birth death process. Hence:

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

Resolving for M/M/m

$$p_n = \frac{\lambda^n}{n!\,\mu^n}\,p_0 \quad \text{for n = 0, 1, 2, … m-1}$$

$$p_n = \frac{\lambda^n}{m!\,m^{n-m}\,\mu^n}\,p_0 \quad \text{for n = m, m+1, … } \infty$$

With traffic intensity

- $\rho=\lambda/(m\mu)$

$$p_n = \frac{(m\rho)^n}{n!} p_0 \text{ for n = 0, 1, 2, … m-1}$$

$$p_n = \frac{\rho^n m^m}{m!} p_0 \text{ for n = m, m+1, … } \infty$$

M/M/m queue

- The rest of the results for this system can be derived from the state probabilities (see in the text book)

m times M/M/1 vs M/M/m

- What is better?
  - m queues of the form M/M/1 with arrival rate λ/m
  - a single system of the form M/M/m with arrival rate λ
- In general, M/M/m will be better because it leads to less waiting time (jobs waiting in a queue do not benefit if a server in another queue is free)

M/M/m/B

- The queuing system has limited buffer capacity. After B buffers are full, jobs are no longer admitted
- State transition diagram is similar to that of M/M/m but it finishes with B (as opposed to having ∞ states)
- As before

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

State probabilities

$$p_n = \frac{\lambda^n}{n!\,\mu^n}\,p_0 = \frac{(m\rho)^n}{n!}\,p_0$$

for n < m

$$p_n = \frac{\lambda^n}{m!\,m^{n-m}\,\mu^n}\,p_0 = \frac{\rho^n m^m}{m!}\,p_0$$

for n = m, m+1, …, B

M/M/m/B

- All the other parameters can be computed from these probabilities (see the textbook)
- Effective arrival rate:
  - Arrival rate λ
  - After B jobs, no more jobs enter the system
  - $\lambda' = \lambda (1-p_B)$ effective arrival rate
  - $\lambda - \lambda' = \lambda p_B$ packet loss rate
- Apply effective arrival rate to Little's Law

Operational Laws

Operational laws

- Operational laws are relationships that apply to certain measurable parameters in a computer system
- They are independent of the distribution of arrival times or service rates
- The parameters are observed for a finite time T_i, yielding operational quantities:
  - Number of arrivals A_i
  - Number of completions C_i
  - Busy time B_i

Job flow balance

- The job flow balance assumption implies:
- Number of arrivals A_i (jobs arriving during T_i)
- Number of completions C_i (jobs completed during T_i)

$$A_i = C_i$$

- Correct use heavily depends on semantics of "jobs completed"

Derivations

- Arrival rate $\lambda_i = A_i / T_i$
- Throughput $X_i = C_i/T_i$
- Utilization $U_i = B_i/T_i$
- Mean Service Time $S_i = B_i/C_i$
- These are variables that change from observational period to observational period
- Some relationships hold for each observational period

Utilization Law

$$U_i = \frac{B_i}{T_i} = \frac{C_i}{T_i} \times \frac{B_i}{C_i} = X_i \cdot S_i$$

The utilization law is simply based on the observation that the service time indicates how long the server needs to complete a job. Hence, the utilization of the server is the number of jobs completed multiplied the time that it takes to complete each one of them

Example Utilization Law

- A disk serves 40 requests per second. Each request requires 0.0225 seconds
- Utilization: 40 x 0.0225 = 90%

- Instrumentation shows that a disk is busy 80% of the time when an application runs. We know that the application produces 20 requests per second
- The time taken per request is 40 ms

Forced Flow Law

- Assume a closed system several devices are connected as a queuing network. The number of completed jobs is C_0
- Assume job flow balance
- If each job makes V_i (visit ratio) requests of the i_{th} device, then $C_i = C_0 V_i$

$$X_i = \frac{C_i}{T_i} = \frac{C_i}{C_0} \times \frac{C_0}{T_i} = XV_i$$

Bottleneck device

- Utilization of the device

$$U_i = X_i S_i = X V_i S_i$$

With the demand for the device $D_i = V_i S_i$

$$U_i = X D_i$$

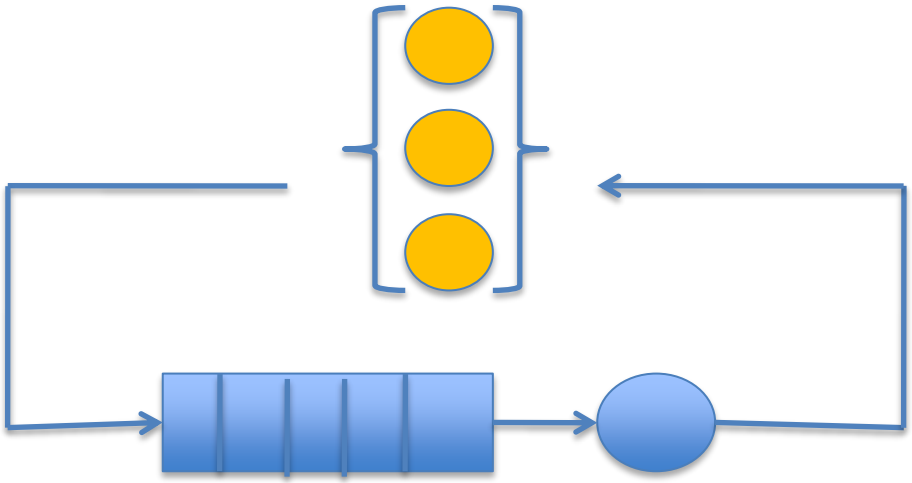
The device with the highest demand is the bottleneck device

Little's Law

- Assuming job flow balance:
- R_i is the response time of the device
- Q_i is the number of jobs in the device
- $\lambda_i = X_i$ (arrival rate equals throughput)

$$Q_i = \lambda_i R_i = X_i R_i$$

Interactive Response Time Law



- Users submit a request, when they get a response, the think for a time Z, and submit the next request

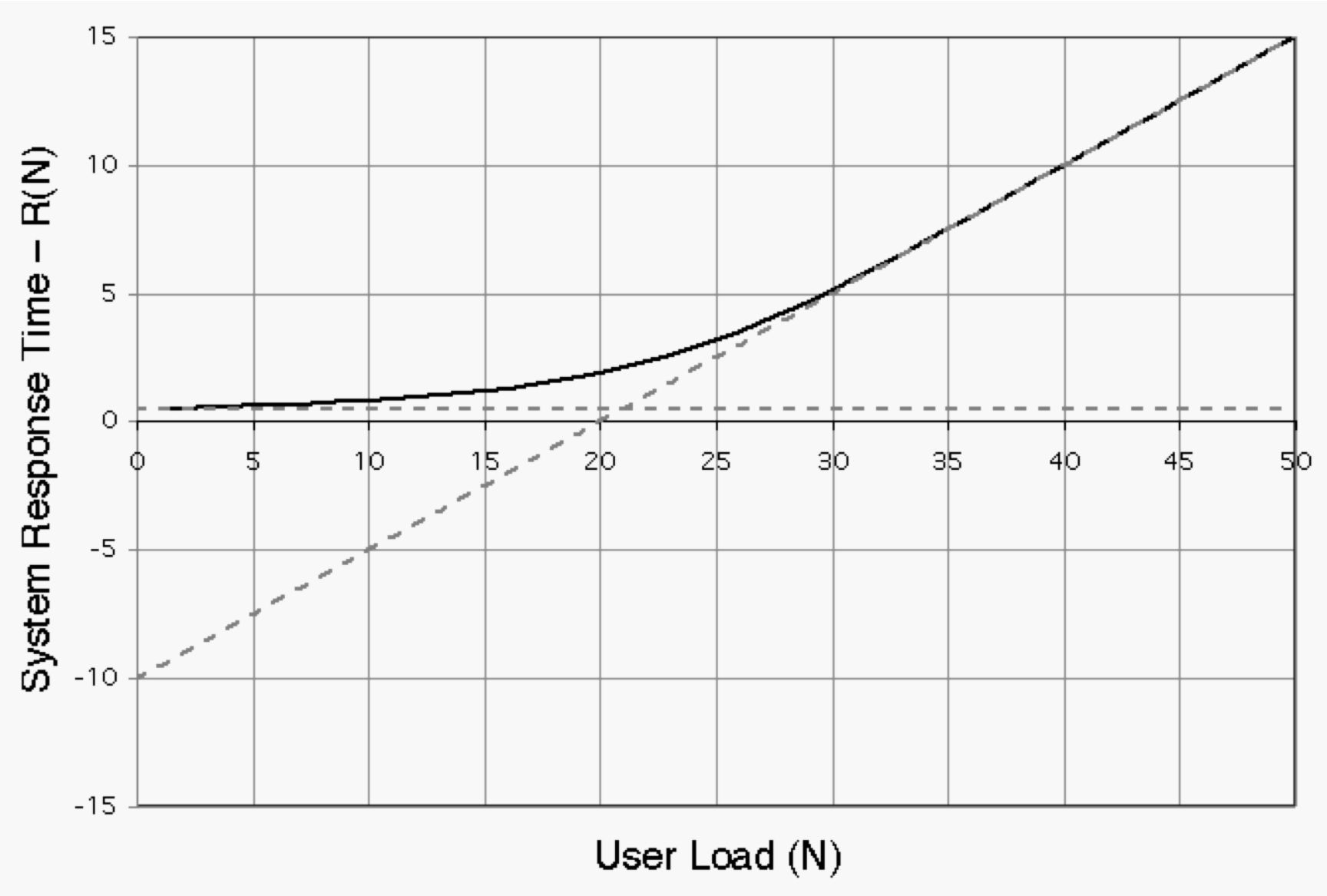
Interactive, closed systems

- Response time R
- Total cycle time R+Z
- Each user generates T/(R+Z) requests in time T
- There are N users

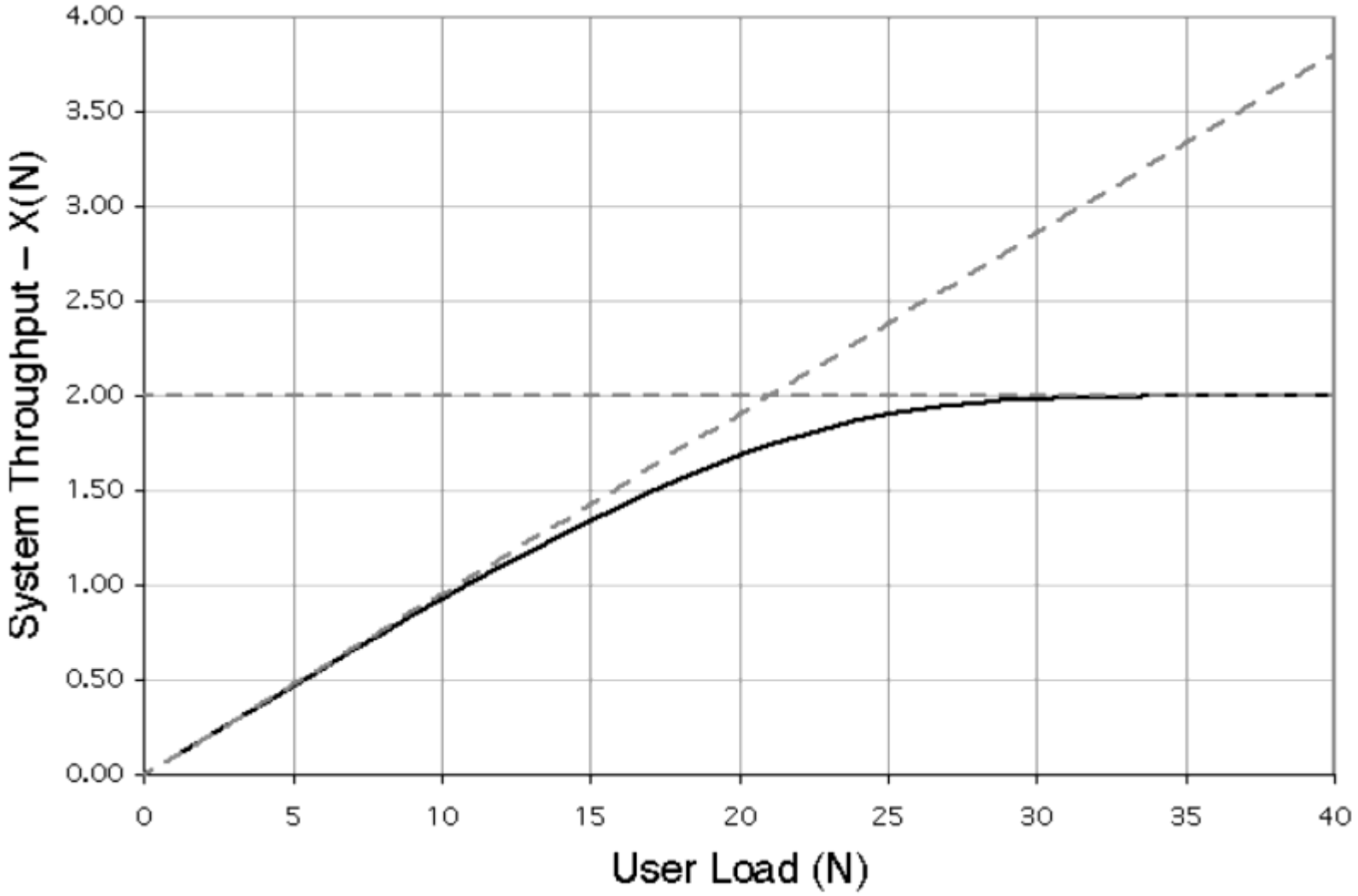
$$X = \frac{Jobs}{Time} = \frac{N\dfrac{T}{R+Z}}{T} = \frac{N}{R+Z}$$

$$R = \frac{N}{X} - Z$$

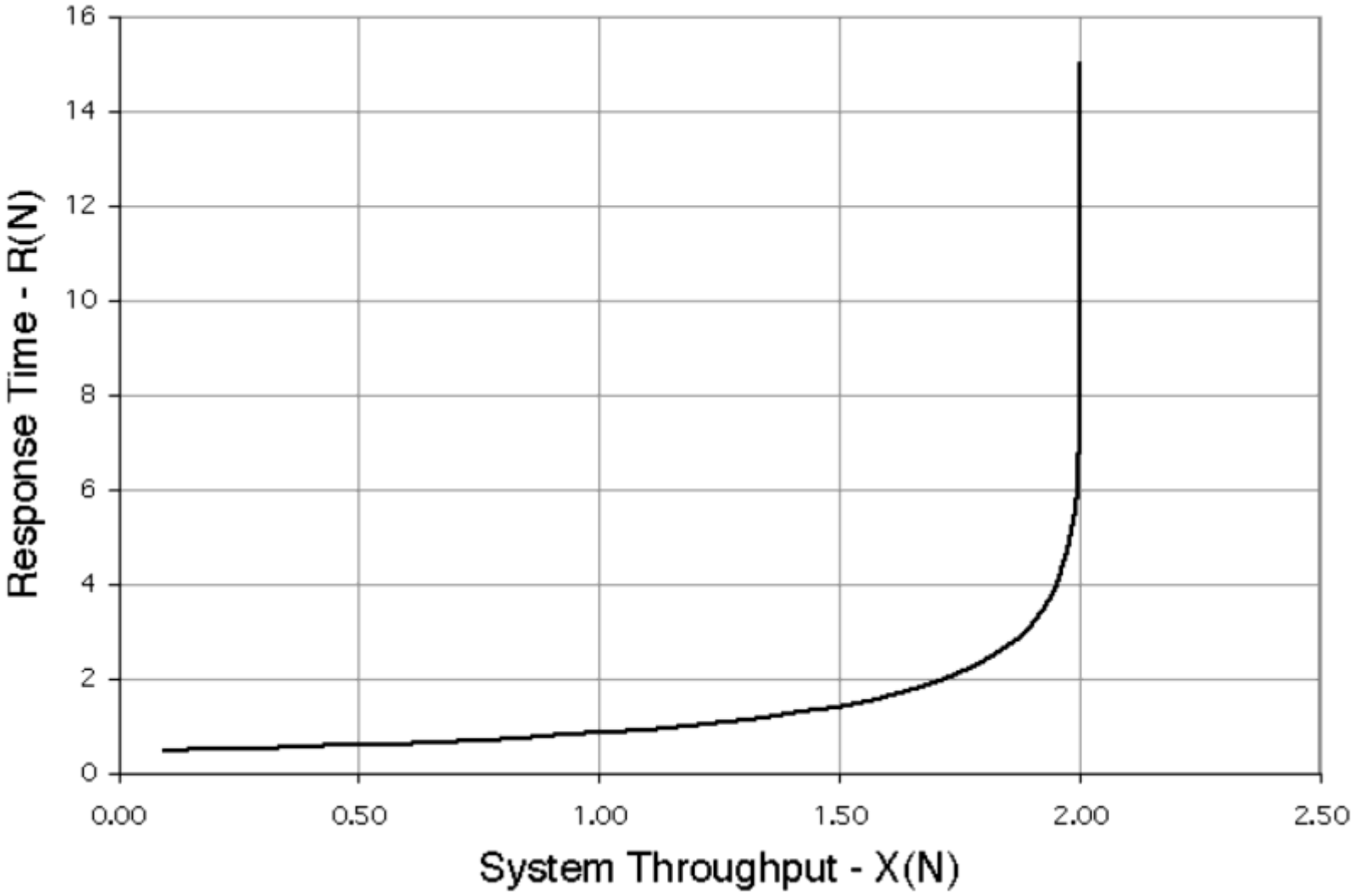
Interactive Law in practice



Interactive law in practice



Interactive law in practice



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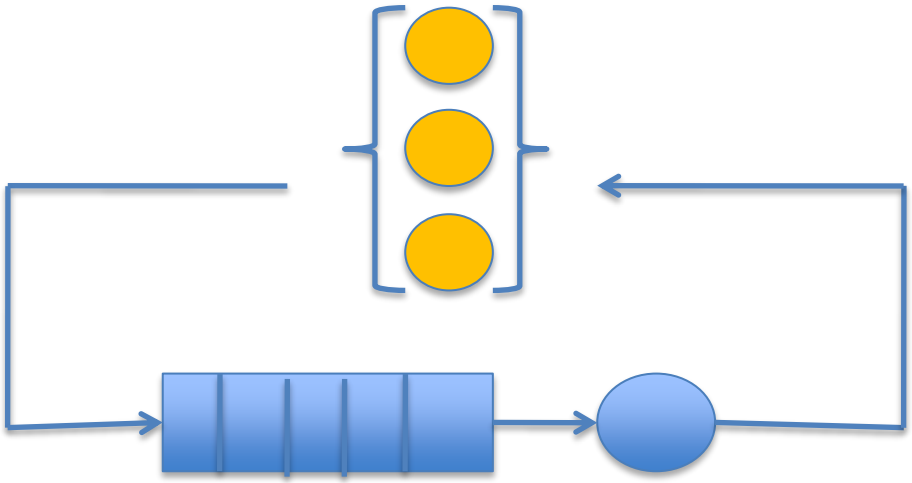

- Real systems are not ideal
  - Network effects
  - Processing overhead(not only wait)
  - Exceptions and not “normal” return values