Advanced Systems Lab
Tutorial IV
Queuing Systems

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Queuing system

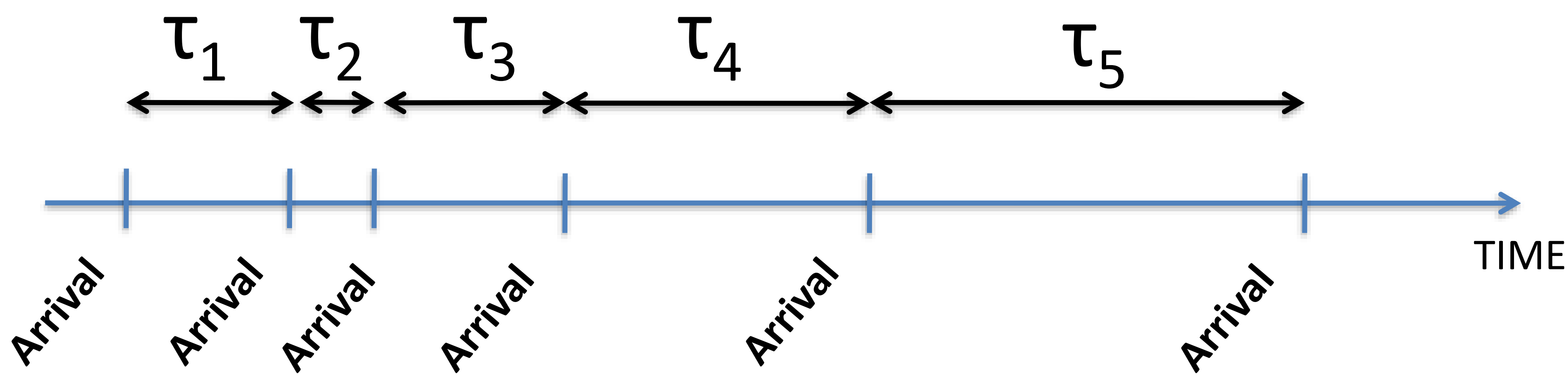
Characterizing a queuing system

- Arrival rate
- Service time
- Service discipline
- System capacity
- Number of servers
- Population size

- A/S/m/C/P/SD
  - A = arrival distribution
  - S = service distribution
  - m = number of servers
  - C = buffer capacity
  - P = population size (input)
  - SD = service discipline
- Notation not standardized

ARRIVAL RATE DISTRIBUTION

Interarrival time



- The interarrival times are assumed to form a sequence of Independent and Identically Distributed (IID) random variables
- Common assumption is a Poisson distribution

Mean arrival rate

- Mean interarrival time = E[τ]
- Mean arrival rate: $\lambda = 1 / E[\tau]$
- λ is not a random variable!
- Examples;
  - a single client submits a query every 200 ms, then λ is 5 queries/second
  - 10 clients submit a query each every 500 ms, then λ is 20 queries/second
  - These are the queries <u>submitted</u> to the system

Assumptions

- Queuing systems assume an arrival rate
  - state independent (does not depend on number of previous jobs)
  - stationary (does not change in time)
- These assumptions do not hold in real systems
  - Burstiness / batch jobs
  - Flash crowds (popularity)
  - Social effects (time of day variant load)

SERVICE RATE DISTRIBUTION

Service time per job

- The time it takes to process a job (only the time it takes to process it, not including the time it has been waiting in the queue) = s
- Mean service rate: $\mu = 1/E[s]$
- If there are *m* servers, mean service rate is $m\mu$
- μ is not a random variable!
- Example:
  – Printer takes on average 20 seconds per job, then $\mu = 0.05$ jobs/second = 3 jobs/minute

Throughput

- Sometimes μ is called the system's throughput
- Careful with the notion of throughput
- This is correct only in some cases
  - There are always jobs ready when a job is finished
  - No overhead in switching to new job
  - All jobs complete correctly
  - Service rate is state independent (does not depend on the number of jobs in the queue)
  - Service rate is stationary (does not change with time)

SERVICE DISCIPLINE

Queue discipline

- FCFS = First Come – First Served
  - Ordered queue
- LCFS = Last Come – First Served
  - Stack
- RR = Round Robin
  - CPU allocation to processes
- RSS = random
- Priority Based

OTHER PARAMETERS

System capacity

- The system (or buffer) capacity is the maximum number of jobs that can be waiting for service
- System capacity includes jobs waiting and jobs receiving service
- In reality = finite
- Analysis = assume infinite capacity
- Finite buffers <u>very important</u> in practice

Number of Servers

- The service can be provided by one or more servers
- Assume work in parallel and independent
- Servers do not interfere with each other
- Total service rate is aggregation of each individual service rate

Population

- The total number of potential jobs that can be submitted to the system:
- Analysis = assume infinite
- In practice:
  - Very large (assume infinite), e.g., number of clicks on a page
  - Finite, number of homework submissions for this lecture
  - Closed systems (output determines input)

GENERAL RESULTS

G/G/1 G/G/m

Offered load

- The offered load or traffic intensity is

$$\rho = \lambda/(m\mu) = \lambda \cdot E[s]/m$$

- The system is stable if

$$\rho < 1 \Rightarrow \lambda < m\mu$$

- In other words, the system is stable if the mean arrival rate (λ) is less than the mean service rate ($m\mu$), otherwise the queue grows without bounds

$\rho = 1$

- Unless arrivals and service are deterministic and exactly scheduled, ρ =1 does not lead to a steady system
  - Randomness prevents queuing from emptying
  - Server cannot catch up
  - Queue grows without bound
- One way to avoid this is <u>flow control</u> (drop jobs when load to high)

Examples

Instrumentation shows that a disk is serving 50 I/O operations per second and the average I/O time is 10 ms. What is the disk utilization?

$$\rho = \lambda \cdot E[s]/m, \text{ with } m = 1$$

$$\rho = 50 \times 0.010 = 0.5 = 50\%$$

Application A generates about 50 I/O requests/s, if the disk is 85 % utilized, what is the average time needed for every I/O? ... 17ms

Further examples

We have allocated 60% of the disk to one application. If we want to maintain an average response time for every I/O operations of 12 ms, what is the maximum number of I/O requests per second that the application can generate?

$$0.6 = \lambda \cdot 12 \text{ ms} \Rightarrow \lambda = 50 \text{ req/s}$$

Some more notation

- $n = n_s + n_q$, where
  - n is the number of jobs in the system (queue)
  - n_s is the number of job in the service
  - n_q is the number of jobs waiting for service
- $w = w_q + s$, where
  - w is the total time in the system
  - w_q is the time waiting in the queue
  - s is the time in the service
- These are all random variables

Little's Law

- For the queuing system:

$$E[n] = \lambda \cdot E[w]$$

- For the queue

$$E[n_q] = \lambda \cdot E[w_q]$$

- With

$$E[n] = E[n_q] + E[n_s]$$

$$E[w] = E[w_q] + E[s] = E[w_q] + 1/\mu$$

Example

Instrumentation shows that the average time to respond to a request was 100 ms and the server received about 100 requests/second. If each active request requires 5 KB of memory, how much memory needs to be reserved for the average number of requests in the system?

Jobs in the system = $100 \cdot 0.1 = 10$ jobs (Little's)

Memory needed = $10 \cdot 5$ KB = 50 KB

Jobs in service

- Using Little's Law, one can derive:

$$E[n_s] = \lambda/\mu = \lambda \cdot E[s]$$

- For a queuing system with m servers

$$E[n_s] = m \cdot \rho$$

that is, the average number of jobs in service is m times the arrival rate divided by the mean service rate

BIRTH-DEATH PROCESSES

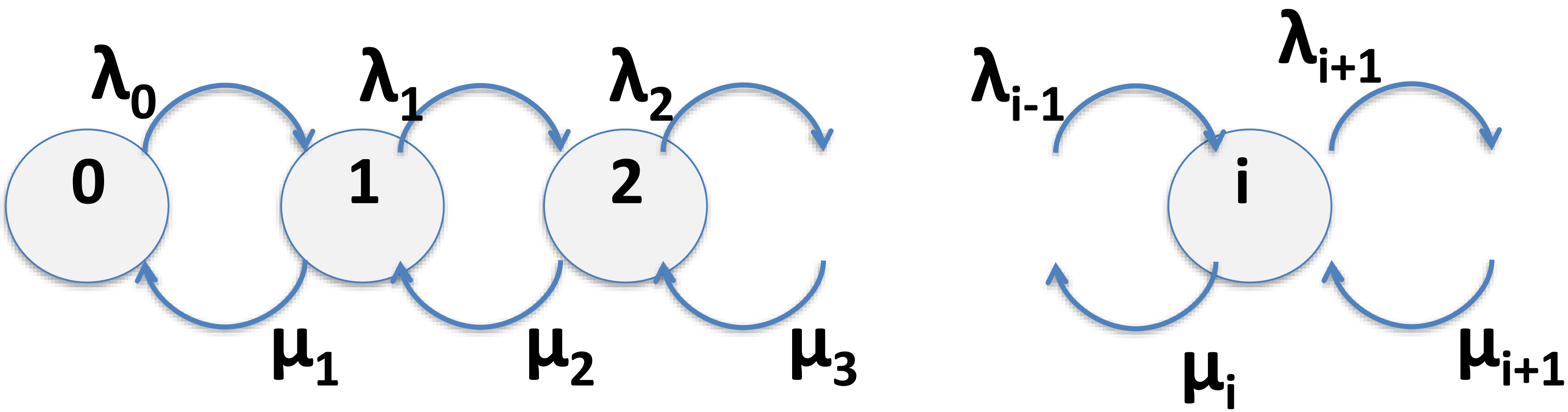
Stochastic processes

- Many of the values in a queuing system are random variables function of time (e.g., the waiting time at a queue)
- Such random functions of times are called stochastic processes
- If the values a process can take are finite or countable, it is a discrete process or a stochastic chain

Markov Processes

- If the future states of a process depend only of the current state and not on past states, the process is a Markov process
- Discrete Markov processes are Markov chains
- A Markov chain in which the transition between states is limited to neighboring states is called a birth-death process

Steady state probability



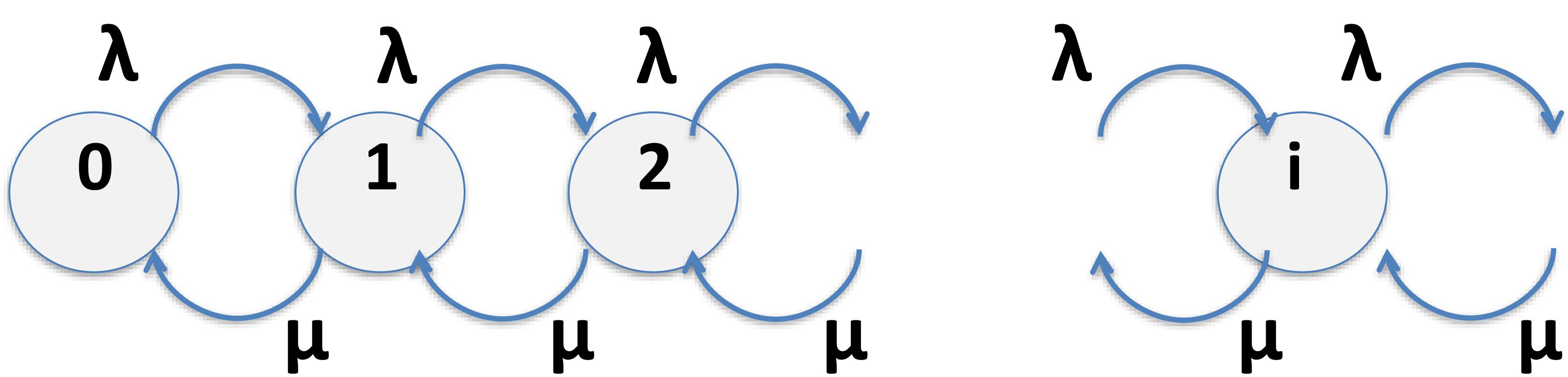
- Probability of being in state n is:

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

M/M/1

M/M/1

- Memoryless distribution for arrival and service
- Single server
- Infinite buffers and FCFS
- Mean arrival rate: λ
- Mean service rate: μ



Basics M/M/1

- From the state probability of a birth-death process:

$$p_n = (\lambda/\mu)^n \, p_{0,} \;\; n = 1,2,\ldots,\infty$$

- or

$$p_n = \rho^n \, p_0$$

- Since all probabilities must add to 1

$$p_0 = 1 - \rho \qquad \text{and} \qquad p_n = (1 - \rho)\, \rho^n$$

Utilization

- Utilization: probability that there is one or more jobs in the system

$$U = 1 - p_0 = \rho$$

M/M/1 behavior

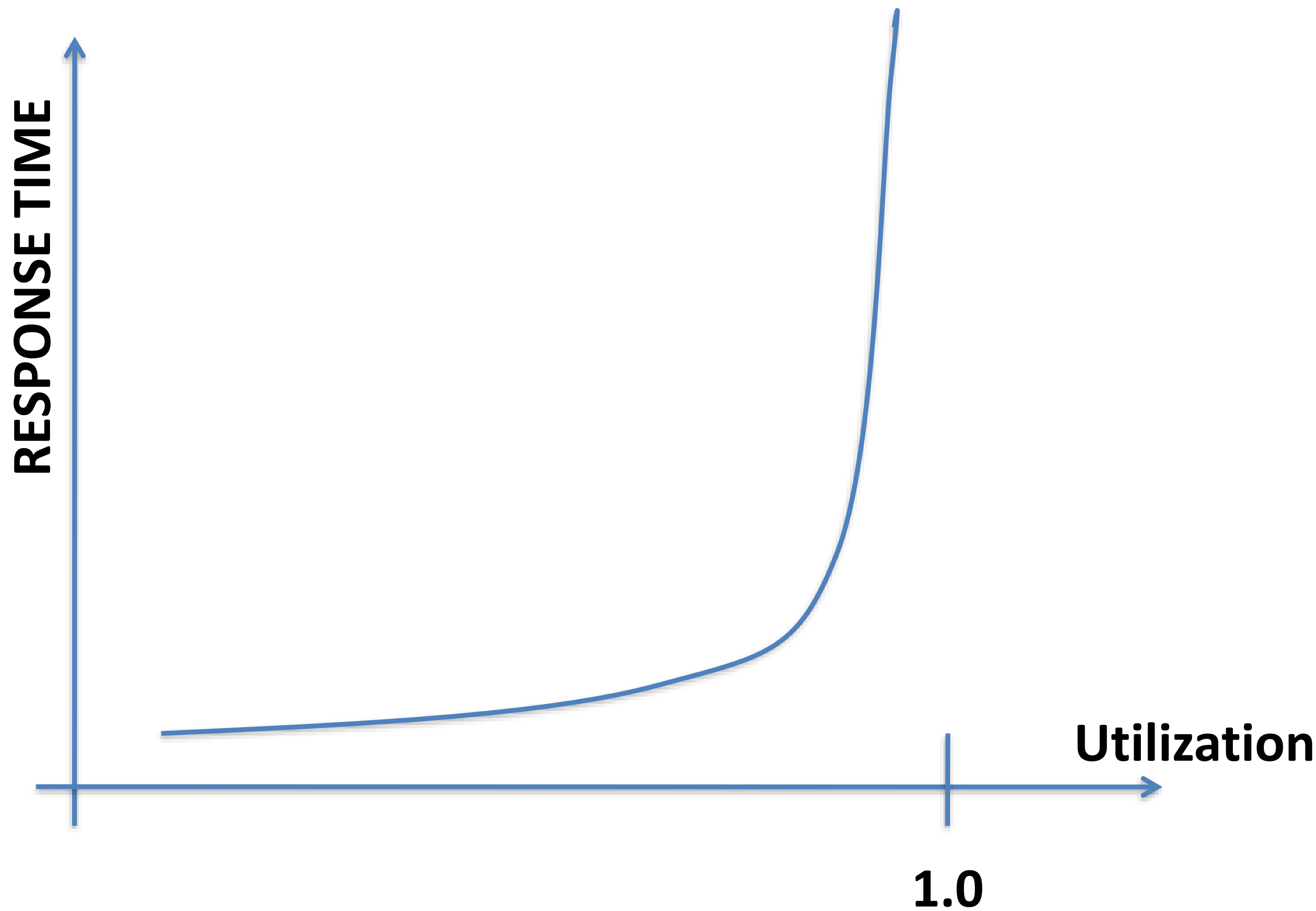
- The mean number of jobs E[n] is

$$E[n] = \sum_{n=1}^{\infty} n \cdot p^n = \sum_{n=1}^{\infty} n(1-\rho)\rho^n = \frac{\rho}{1-\rho}$$

- Applying Little's Law we get the response time

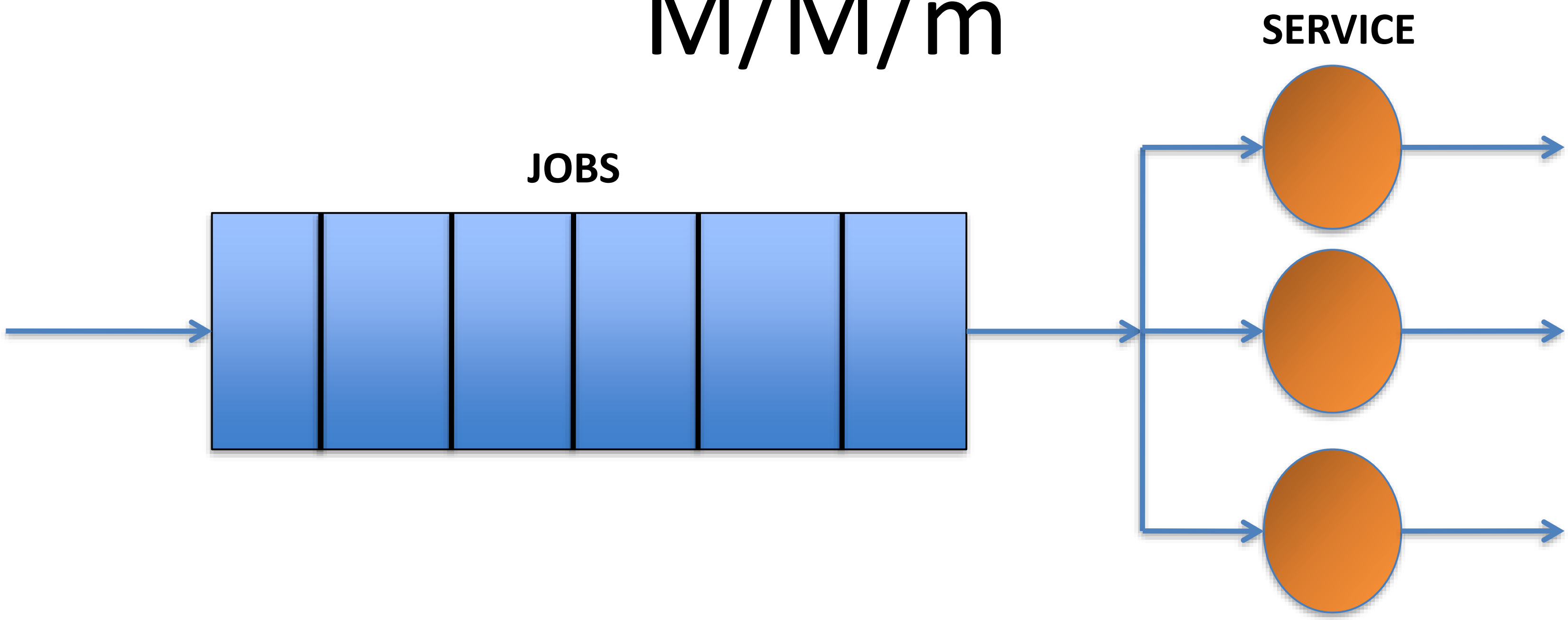
$$E[w] = \frac{1/\mu}{1-\rho}$$

Response time in M/M/1



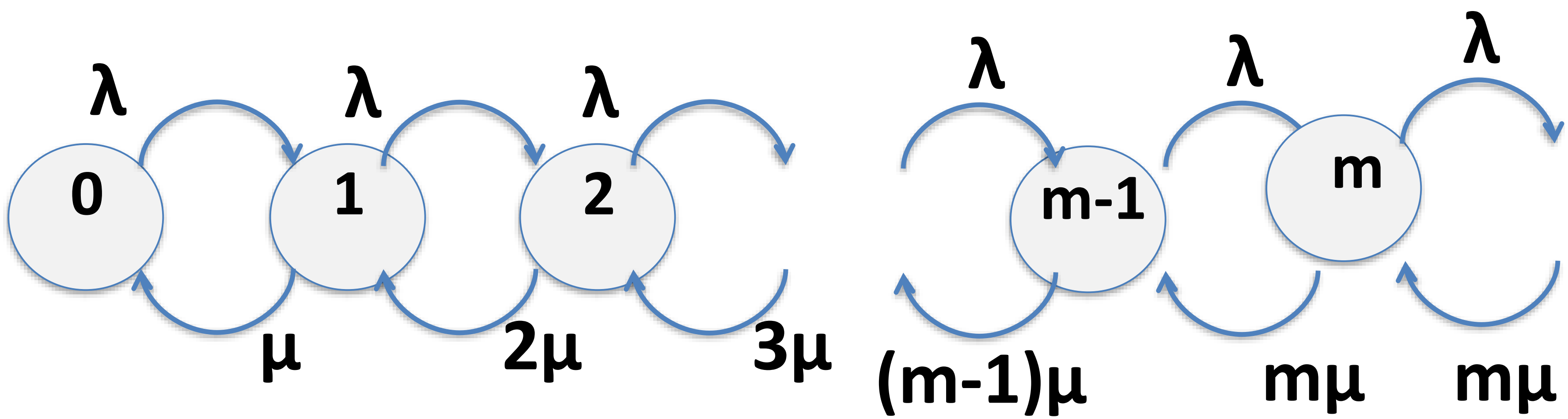
M/M/m and M/M/m/B

M/M/m



- Each server serves μ jobs per unit of time
- Jobs get service right away if less than m jobs in system, otherwise they wait in queue.

Probabilities in M/M/m



- Number of jobs in a M/M/m system is a birth death process. Hence:

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

Resolving for M/M/m

$$p_n = \frac{\lambda^n}{n!\,\mu^n} p_0$$ for n = 0, 1, 2, … m-1

$$p_n = \frac{\lambda^n}{m!\,m^{n-m}\mu^n} p_0$$ for n = m, m+1, … ∞

With traffic intensity

- ρ=λ/(mμ)

$$p_n = \frac{(m\rho)^n}{n!} p_0 \text{ for n = 0, 1, 2, … m-1}$$

$$p_n = \frac{\rho^n m^m}{m!} p_0 \text{ for n = m, m+1, … } \infty$$

M/M/m queue

- The rest of the results for this system can be derived from the state probabilities (see in the text book)

m times M/M/1 vs M/M/m

- What is better?
  - m queues of the form M/M/1 with arrival rate λ/m
  - a single system of the form M/M/m with arrival rate λ
- In general, M/M/m will be better because it leads to less waiting time (jobs waiting in a queue do not benefit if a server in another queue is free)

M/M/m/B

- The queuing system has limited buffer capacity. After B buffers are full, jobs are no longer admitted
- State transition diagram is similar to that of M/M/m but it finishes with B (as opposed to having ∞ states)
- As before

$$p_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} p_0$$

State probabilities

$$p_n = \frac{\lambda^n}{n!\,\mu^n} p_0 = \frac{(m\rho)^n}{n!} p_0$$

for n < m

$$p_n = \frac{\lambda^n}{m!\,m^{n-m}\mu^n} p_0 = \frac{\rho^n m^m}{m!} p_0$$

for n = m, m+1, …, B

M/M/m/B

- All the other parameters can be computed from these probabilities (see the textbook)
- Effective arrival rate:
  - Arrival rate λ
  - After B jobs, no more jobs enter the system
  - $\lambda' = \lambda (1-p_B)$ effective arrival rate
  - $\lambda - \lambda' = \lambda p_B$ packet loss rate
- Apply <u>effective arrival rate</u> to Little‘s Law