Queuing Networks, MVA, Bottleneck Analysis

Advanced Systems Lab

November 16, 2017
Network of Queues

Last Week:
We used M/M/1 and M/M/m to model the whole system/middleware.

This week:
We build a network of queues consisting of multiple queues, each representing a component of the system. The more detailed your network of queues is, the more accurate is the resulting model.
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We build a network of queues consisting of multiple queues, each representing a component of the system. The more detailed your network of queues is, the more accurate is the resulting model.
Operational Laws

- Utilization Law
- Forced Flow Law
- Little’s Law
- Interactive Response Time Law

Notation:
- $T$ Observation interval (time)
- $C$ Number of completions
- $A$ Number of arrivals
- $B$ Busy time
- $X$ Throughput

Type of devices:
- Fixed-capacity service center, for instance $M/M/1$
- Delay center, no queuing, response time independent of load, infinite capacity, can also be modeled as $M/M/\infty$
- Load-dependent service center, for instance $M/M/m$
Utilization Law

Utilization of a device $i$, $U_i$, can be calculated as follows:

$$U_i = \frac{B_i}{T} = \frac{C_i}{T} \times \frac{B_i}{C_i}$$

or

$$U_i = X_i \times S_i$$
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or

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Example:
Pizza chef bakes a pizza in: 6 min
Every hour 7 customers enter the shop and order 1 pizza each
**Throughput:** 7 pizzas/hour
**Service time:** 0.1 hour
**Utilization:** $U_i = X_i \times S_i = 7 \times 0.1 = 0.7$
Forced Flow Law (I)

The Forced Flow law holds if the number of job arrivals $A_i$ at each device is the same as the number of job completions $C_i$. \[ A_i = C_i \]

$V_i$ is the visit ratio to the device $i$:

\[ V_i = \frac{C_i}{C_0} \text{ or } C_i = C_0 \times V_i \]

$C_0$ is the number of jobs leaving the system
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$C_0$ is the number of jobs leaving the system

**Example:**

$C_0$: 10

$C_{\text{pizza}} = C_0 \times V_{\text{pizza}} = 10 \times 0.7 = 7$

$C_{\text{gelato}} = C_0 \times V_{\text{gelato}} = 10 \times 0.3 = 3$
Forced Flow Law (II)

The throughput of device $i$ is:

$$X_i = \frac{C_0}{T} \cdot \frac{C_i}{C_0}$$

$$X_i = X \cdot V_i$$
Forced Flow Law (II)

The throughput of device $i$ is:

$$X_i = \frac{C_0}{T} \ast \frac{C_i}{C_0}$$

$$X_i = X \ast V_i$$

Combining Utilization & Forced Flow Law:

Utilization of device $i$:

$$U_i = X_i \ast S_i = X \ast V_i \ast S_i$$
Forced Flow Law (II)

The throughput of device $i$ is:

$$X_i = \frac{C_0}{T} \cdot \frac{C_i}{C_0}$$

$$X_i = X \cdot V_i$$

Combining Utilization & Forced Flow Law:

Utilization of device $i$:

$$U_i = X_i \cdot S_i = X \cdot V_i \cdot S_i$$

Device with highest utilization $U_i$ is the **bottleneck device**.
Little’s Law

Can be applied to devices

\[ Q_i = \lambda_i \times R_i \]

if the job flow is balanced:

\[ Q_i = X_i \times R_i \]
Interactive Response Time Law

\( N: \) number of users/clients
\( Z: \) think time

Interactive law: \( R = \frac{N}{X} - Z \) or \( X = \frac{N}{R+Z} \)

Example:
There are 25 people in a bar.
It takes 5 min to order and receive a beer.
It takes 15 min to drink the beer.

\( N: 25 \)
\( R: 5 \text{ min} \)
\( Z: 15 \text{ min} \)

Throughput of bartender:
\( X = \frac{25}{15+5} = 1 \text{ beer/min} \)
Interactive Response Time Law

$N$: number of users/clients
$Z$: think time
Interactive law: $R = \frac{N}{X} - Z$ or $X = \frac{N}{R+Z}$

Example:
There are 25 people in a bar.
It takes 5 min to order and receive a beer.
It takes 15 min to drink the beer.
$N$: 25
$R$: 5 min
$Z$: 15 min
Throughput of bartender: $X = \frac{25}{15+5} = 1.25$ beers/min
Mean-Value-Analysis (MVA)

- Iteratively computes the performance for $n \in \{1, N\}$ users/clients
- Applies operational laws
- Allows you to plot graphs based on your model

Use a tool/library to compute the MVA, for instance GNU Octave:
http://www.gnu.org/software/octave/
http://www.moreno.marzolla.name/software/queueing/

The (above) algorithm only works with fix-capacity service centers and delay centers.
The example system is a closed system. The system implements a Queue data structure which supports *PUSH* and *POP* operations. To make it more scalable a middleware layer is introduced. The middleware has for each type of request a queue and corresponding worker(s). *PUSH* requests are forwarded to all backend servers, while *POP* requests are load-balanced among them.
1st Example: Network of Queues (I)

Modeling:
- Network as delay center
- Parsing thread as M/M/1 → $V_{\text{Parse}} = 1$
- PUSH worker as M/M/1 → $V_{\text{PUSH}} = \text{pushratio}$
- POP worker as M/M/1 → $V_{\text{POP}} = 1 - V_{\text{PUSH}}$
- Servers as M/M/1 → $V_{\text{server0}} = V_{\text{PUSH}} + 1$
1st Example: Network of Queues (I)

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- Network as delay center
- Parsing thread as M/M/1 \( \rightarrow V_{Parse} = 1 \)
- PUSH worker as M/M/1 \( \rightarrow V_{PUSH} = \text{pushratio} \)
- POP worker as M/M/1 \( \rightarrow V_{POP} = 1 - V_{PUSH} \)
- Servers as M/M/1 \( \rightarrow V_{server0} = V_{PUSH} + \frac{1}{3} V_{POP} \)
1st Example: Network of Queues (II)

Parameter:
Push ratio: 0.3

Measured/Derived:
Throughput $X$ of the system: 100 req/s
Service time $S_i$ for each device
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Calculate throughput for each device: $X_i = X \times V_i$

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Calculate utilization for each device: $U_i = X_i \ast S_i$

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Let’s change our example system and correspondingly the model by adding a second POP worker and POP queue. Now we are modeling a different system.
2nd Example: Network of Queues (I)

Modeling:
- Network as delay center
- Parsing thread as M/M/1
- PUSH worker as M/M/1
- POP workers as two M/M/1
- Servers as M/M/1

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2nd Example: Network of Queues (I)

Modeling:
- Network as delay center
- Parsing thread as M/M/1 $\rightarrow V_{Parse} = 1$
- PUSH worker as M/M/1 $\rightarrow V_{PUSH} = \text{pushratio}$
- POP workers as two M/M/1 $\rightarrow V_{POP0} = \frac{1 - V_{PUSH}}{2}$
- Servers as M/M/1 $\rightarrow V_{server0} = V_{PUSH} + \frac{1}{3} V_{POP0} + \frac{1}{3} V_{POP1}$
2nd Example: Network of Queues (II)

Parameter:
Push ratio: 0.3

Measured/Derived:
Throughput $X$ of the system: 100 req/s
Service time $S_i$ for each device
2nd Example: Network of Queues (II)

**Parameter:**
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2nd Example: Network of Queues (II)

Parameter:
Push ratio: 0.3

Measured/Derived:
Throughput $X$ of the system: 100 req/s
Service time $S_i$ for each device

Calculate throughput for each device: $X_i = X \times V_i$

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Let’s adapt our example system again. Now the two POP workers are going to share the same POP queue. This is again a different system.
3rd Example: Network of Queues (I)

Modeling:
- Network as delay center
- Parsing thread as M/M/1 $\rightarrow V_{\text{Parse}} = 1$
- PUSH worker as M/M/1 $\rightarrow V_{\text{PUSH}} = \text{pushratio}$
- POP workers as M/M/2 $\rightarrow V_{\text{POP}} = 1 - \text{pushratio}$
- Servers as M/M/1 $\rightarrow V_{\text{server0}} = V_{\text{PUSH}} + 1$
3rd Example: Network of Queues (I)

Modeling:
- Network as delay center
- Parsing thread as M/M/1 $\rightarrow V_{Parse} = 1$
- PUSH worker as M/M/1 $\rightarrow V_{PUSH} = \text{pushratio}$
- POP workers as M/M/2 $\rightarrow V_{POP} = 1 - V_{PUSH}$
- Servers as M/M/1 $\rightarrow V_{server0} = V_{PUSH} + \frac{1}{3} V_{POP}$
3rd Example: Network of Queues (II)

**Parameter:**
- Push ratio: 0.3

**Measured/Derived:**
- Throughput $X$ of the system: 100 req/s
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M/M/2 cannot be modelled as fixed-capacity device, use extended MVA instead.
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Lesson learned

In the previous example we saw the limitations of the operation laws and the basic MVA algorithm.

As we have seen last week: $n$ M/M/1 are not equal to one M/M/n. Depending on your actual system one of them is more suitable. M/M/m devices can be modeled with extended MVA.
Office hours

Office hours are indented to provide you advice that will help you to complete the project and the report. To make an appointment, contact your teaching assistant by email.

- Make sure you come prepared with concrete and well formulated questions. If possible, include them in your email.
- We will not complete the assignment for you and neither recommend nor make design decisions on your behalf.
- We will not debug your code, provide technical support for your setup/scripts/data analysis, or give hints about whether what you have done so far is enough.
- We will not grade your project in advance, so please avoid questions that try to determine whether what you have done is correct or sufficient for a passing grade.

Days & Times can be found on the website.
The spending is now correctly reported on https://www.microsoftazuresponsorships.com, please check it there.